Finite Automata with Output

Models of Computation
Lecture #11
Chapter 8

Extending capabilities

Machines that can produce output
- note that an FA can represent output via a state, but we now mean explicit output

Two related models
- Moore machine
- Mealy machine

Moore machine

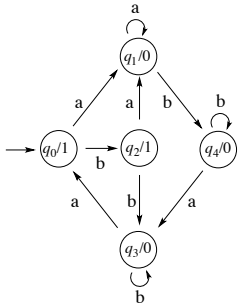
- a finite set of states q_0, q_1, q_2, … where q_0 is the start state
- an input alphabet with letters in Σ
- an output alphabet with characters in Γ
- a transition table with a transition for each input letter for each state
- an output table that shows what character to print as each state is entered

Moore machine

Compared to FA
- Start state indicated by an arrow not a –
- There is no final state
- Doesn't define a language explicitly
- In the state, include an indication of the letter to be printed on entering the state
- By definition the start state character is printed at the beginning because we "enter" that state to begin a run

Moore machine, example

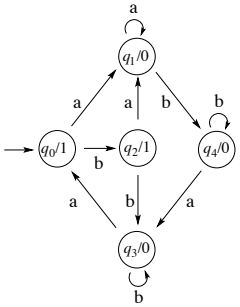
- Σ = { a b }
- Γ = { 0 1 }
- states: q_0, q_1, q_2, q_3, q_4



Moore machine, example

Input string:
abbaabba

Output printed:
100001101



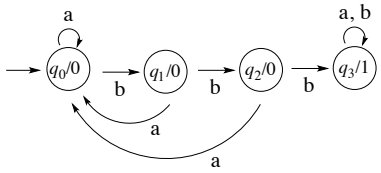
Moore machine, "Acceptor"

1. Design a machine which prints a special character only when entering a final state.
2. Run the machine.
3. If the output ends with the special character then the input was "accepted" by the machine.

Moore machine, "Acceptor"

Example: accept any input with a bbb substring.

When run on an input, if the output ends in 1 then the string is accepted.

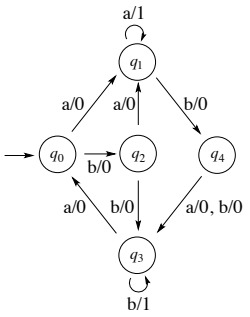


Alternate model: Mealy machine

- a finite set of states q_0, q_1, q_2, … where q_0 is the start state
- an input alphabet with letters in Σ
- an output alphabet with characters in Γ
- a transition table with a transition for each input letter for each state, with output character to be printed when traveling the edge

Mealy machine, example

- Σ = { a b }
- Γ = { 0 1 }
- states: q_0, q_1, q_2, q_3, q_4



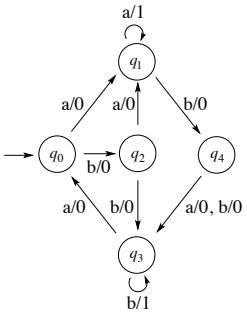
Mealy machine, example

Input string:
aaaabbbbaabb

Output printed:
011100110000

What does it do?

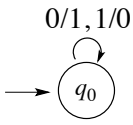
How man times did we loop? Count the 1's.



Design problem 1

Design a Mealy machine to transform an input in binary into it's 1's complement

(very simple machine!)



Design problem 2

Design a Mealy machine to increment a binary number when input least significant digit first.

For example:

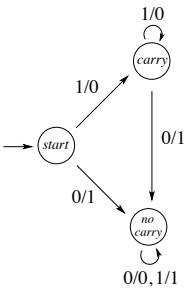
0000 → 1000
1011 → 0111
1111 → 0000 (overflow)

Design problem 2

Increment
Little end in

For example:

0000 → 1000
1011 → 0111
1111 → 0000 (overflow)



Which model is more powerful?

Theorem:
Moore = Mealy
ignoring initial character from Moore machine

Proof:
Part 1. Show for every Moore, there is a Mealy that is equivalent
Part 2. Show for every Mealy, there is a Moore that is equivalent

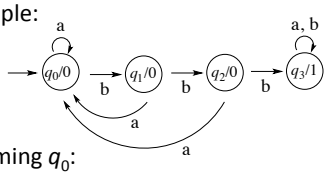
Moore = Mealy, Part 1

Constructive algorithm, Moore → Mealy

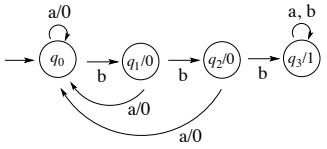
1. Consider any specific state in a Moore machine
2. State contains a print instruction
3. Append the print character to each incoming edge and delete the print character from the state.
4. Repeat for all states.

Moore = Mealy, Part 1

For example:

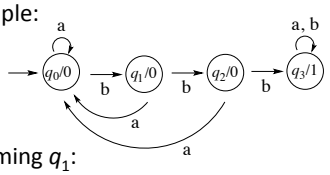


Transforming q_0:

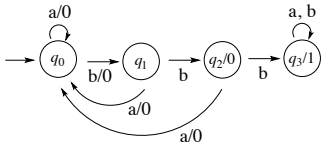


Moore = Mealy, Part 1

For example:

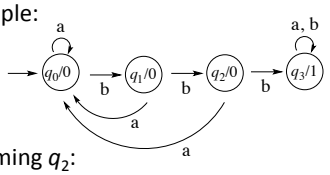


Transforming q_1:

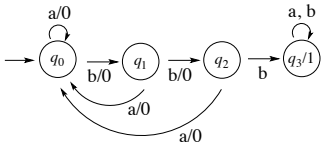


Moore = Mealy, Part 1

For example:

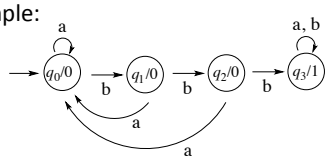


Transforming q_2:

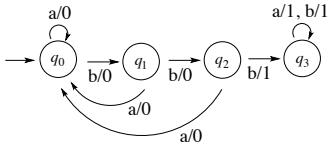


Moore = Mealy, Part 1

For example:



Finally:



Moore = Mealy, Part 2

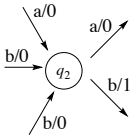
Constructive algorithm, Mealy → Moore

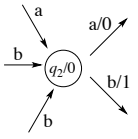
1. Consider any specific state in a Moore machine
2. Create "twin" states if incoming edges have different output symbols
3. Append the print character to each state now that incoming edges always have the same printing instruction; delete the print character from the edge.
4. Repeat for all states.

Moore = Mealy, Part 2

Examples.

Incoming edges all print same character

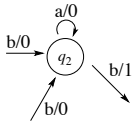


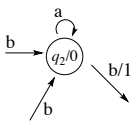


Moore = Mealy, Part 2

Examples.

Incoming edges with a loop? No different.

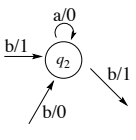


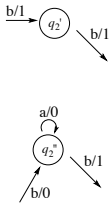


Moore = Mealy, Part 2

Examples.

Incoming edges with different print characters lead to twin states.

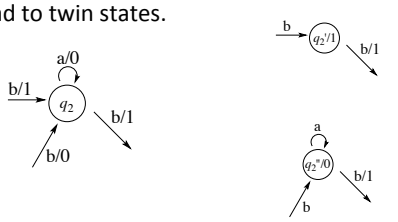




Moore = Mealy, Part 2

Examples.

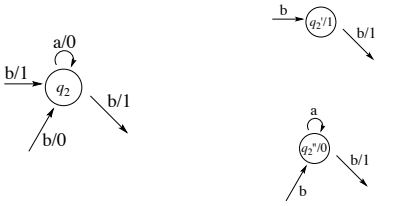
Incoming edges with different print characters lead to twin states.



Moore = Mealy, Part 2

Examples.

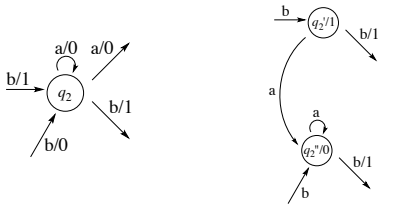
But, what to do with the "a" transition from q_2' ?



Moore = Mealy, Part 2

Examples.

Send it to q_2'' since that's how you print the 0.

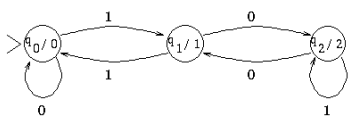


Mo(o)re Examples

- modulo 3 calculator
- MUL 2 calculator

Mo(o)re Examples

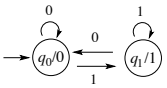
- modulo 3 calculator



http://pearl.ics.hawaii.edu/~sugihara/course/ics241/notes/11-08n20.html#53

Mo(o)re Examples

- MUL 2 calculator



Vending Machine – Meal(y) time

- 20 cent vending machine