MAT342/CPS342 Numerical Analysis

Spring 2019

General Information

Meeting Time and Place

Monday, Wednesday, and Friday: 2:10–3:10 p.m., KOS 126.

Professor

Dr. Jonathan Senning, 246 Ken Olsen Science Center
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Office Hours

Monday, Wednesday: 3:20–4:20 p.m.,
Tuesday and Thursday: 1:30–3:00 p.m.,
and by appointment.

Text & Other Materials

Numerical Mathematics and Computing, 7th Edition, W. Cheney and D. Kincaid, Brooks/Cole, 2013.

Online materials associated with this class can be found on the departmental web server at *http://www.math-cs.gordon.edu/courses/mat342*. Grades will be maintained using Blackboard.

Prerequisite

Satisfactory completion of two semesters of calculus and the ability to write simple computer programs with conditionals and looping structures. Students will benefit from already having taken MAT225 Differential Equations and/or MAT232 Linear Algebra.

Accommodations

Our academic community is committed to providing access to a Gordon education for students with disabilities. A student with a disability who intends to request academic accommodations should follow this procedure:

1. Meet with a staff person from the Academic Success Center (ASC) and provide them with current documentation of the disability.
2. Obtain a Faculty Notification Form from the ASC, listing appropriate accommodations.
3. Submit this form to professors and discuss those accommodations with them, ideally within the first two weeks of classes.

Some accommodations need more time to arrange so communicating early in the semester is important. For more information consult http://www.gordon.edu/academicaccessibility or email *asc@gordon.edu*.

Academic Dishonesty

Academic dishonesty is regarded as a major violation of both the academic and spiritual principles of this community and may result in a failing grade or suspension. Academic dishonesty includes plagiarism, (see Plagiarism in Student Handbook), cheating (whether in or out of the classroom), and abuse or misuse of library materials when such abuse or misuse can be related to course requirements.

Course Description

Introduction

In many disciplines, certainly in the natural sciences and many social sciences, there exist problems which can be modeled mathematically such that solving the mathematical model requires the solution of differential equations. Aside from simple textbook problems, most of these differential equations must be solved numerically, often by reducing the problem to that of solving one or more linear systems. Other problems also arise frequently: finding the solutions to single variable and multivariable equations, fitting functions to data, generating random numbers, etc.

While the numerical problems we survey in this course are fairly simple, all of topics we will discuss are used in various ways in the solution of much more complicated problems. Some of the applications are easy to see and will be pointed out to you; others are more subtle.

By now you know that what comes out of a computer is only as good as what goes in; actually it might be better to say that what comes out of a computer is *no better* than what goes in. Other difficulties can arise as well. We will see that it is not good enough to simply solve a mathematical problem, we also must justify, to ourselves at the very least but usually to whoever we are solving the problem for, that our answer is correct with a certain degree of accuracy.

Course Content and Learning Outcomes

This course will cover most of Chapters 1–12:

- Chapter 1: Mathematical Preliminaries and Floating-Point Representation
- Chapter 2: Linear Systems
- Chapter 3: Nonlinear Equations
- Chapter 4: Interpolation and Numerical Differentiation
- Chapter 5: Numerical Integration
- Chapter 6: Spline Functions
- Chapter 7: Initial Value Problems
- Chapter 8: More on Linear Systems
- Chapter 9: Least Squares Methods and Fourier Series
- Chapter 10: Monte Carlo Methods and Simulation
- Chapter 11: Boundary Value Problems
- Chapter 12: Partial Differential Equations

Students completing this course will be able to

- effectively use computational tools to solve a variety of mathematical problems, specifically those listed above,

- understand and avoid potential sources of error in numerical methods, and
- quantify the possible error in numerical solutions.

Procedure and Workload Expectations

Class time will primarily be devoted to lecture and discussion. I encourage you to ask questions during class regarding the material presented and at times I may ask you to perform some work during our class meeting times.

For each semester hour of credit, students should expect to spend a minimum of 2–3 hours per week outside of class in engaged academic time. This time includes reading, writing, studying, completing assignments, lab work, or group projects, among other activities.

Course Requirements

Attendance

You are expected to attend class and will be responsible for what transpires in class regardless of your attendance. As a courtesy to others, please avoid arriving late and do not leave during class unless it is an emergency or you have made prior arrangements with me. If you are aware of classes you will need to miss because of field trips, athletic events, or for personal reasons, please let me know in advance.

I expect that during class you will not use your cell phone, tablet or laptop for non-class related conversations or activities. These activities prevent you from fully concentrating on our topic and they are often distracting to those around you.

Homework Assignments

Homework assignments will be assigned throughout the semester and will usually be due the following class period. Late homework will generally not be accepted except in unusual circumstances or by prior arrangement with the instructor. You are expected to attempt all the assigned problems before the next class period. The following are required for all assignments:

- Assignments are to be done on 8.5x11 paper.
- Pages *must not* have ragged edges from spiral bound notebooks.
- Solutions should be laid out in an organized, legible manner and presented in the order that they were assigned.
- Final answers (where appropriate) should be clearly marked, either by highlighting or by enclosing in a neat circle or box.
- Multiple page assignments must be fastened (e.g. stapled but not folded) together.

You are permitted to work together on the homework assignments. However, the work you turn in should be your own. These problems should be considered tools to help you better understand the theory and to become more proficient with the techniques of this course. It is essential that you understand the solution to each problem in order to derive the greatest benefit from this course.

Computer Problems

Throughout the semester you will be problems that require the use of a computer and frequently will involve small amounts of programming. These will vary in complexity and are designed to give you some hands-on experience with the concepts and methods discussed in the class. While all of these exercises could be carried out using programming languages such as C++ or Java, we will use the high-level languages *Python* (http://www.python.org), *Octave* (http://www.octave.org), or MATLAB. These interpreted languages have the advantage that one can issue commands interactively or group commands together to form a program. We will use these languages to "tinker" with the topics we are working on and also to create entire programs designed to solve specific problems.

Term Project

Each student will carry out a research project during the semester. These will be done individually or in teams of two. Ideally this project will allow you to explore an area where numerical computing overlaps another area of interest that you have. You will need to choose a topic and discuss it with me to get my approval during the first four weeks of the semester. These projects should involve programming and numerical experiments as well as have a significant writing component. You will need to locate and utilize references beyond our text, carry out your experimentation and then write a report that explains your work in a coherent manner. The projects will be due two weeks before the end of the semester to allow me time to comment on them and return them to you for revisions. The final version will be due at the end of the semester.

Examinations

There will be a midterm exam and a comprehensive final exam. Any missed exam will receive the grade of zero. You may petition me in writing if you must miss an exam. This should be done as soon as possible, preferably before the exam. If I agree that you have a valid reason to miss the exam during the scheduled hour, I will arrange an alternative time for you to take the exam.

Grading Procedure

Your final average will be computed using the following table:

Component	***Percentage***
Homework	20%
Computer Exercises	20%
Term Project	20%
Midterm	20%
Final	20%

The following table shows the correspondence between the final average and letter grades.

(100 – 96]	A+	(88 – 84]	B+	(76 – 72]	C+	(64 – 60]	D+
(96 – 92]	A	(84 – 80]	B	(72 – 68]	C	(60 – 56]	D
(92 – 88]	A–	(80 – 76]	B–	(68 – 64]	C–	(56 – 52]	D–

Tentative Schedule

Date	***Section(s)***	***Topic***	***Project***
Jan 16, Wed	1.1	Introduction	
Jan 18, Fri	1.2	Mathematical Preliminaries: Taylor Series	
Jan 21, Mon	No Class	Martin Luther King Jr. Birthday	
Jan 23, Wed	1.3	Floating-Point Representation	
Jan 25, Fri	1.4	Loss of Significance	
Jan 28, Mon	2.1	Naïve Gaussian Elimination	
Jan 30, Wed	2.2	Gaussian Elimination with Scaled Partial Pivoting	
Feb 1, Fri	2.3	Tridiagonal and Banded Systems	
Feb 4, Mon	3.1	Bisection Method	
Feb 6, Wed	3.2	Newton's Method	
Feb 8, Fri	3.3	Secant Method	
Feb 11, Mon	4.1	Polynomial Interpolation	Init. Topic
Feb 13, Wed	4.2	Errors in Polynomial Interpolation	
Feb 15, Fri	4.3	Estimating Derivatives and Richardson Extrapolation	
Feb 18, Mon	5.1	Numerical Integration	
Feb 20, Wed	5.1	Trapezoid Method	
Feb 22, Fri	5.2	Romberg Algorithm	
Feb 25, Mon	5.3	Simpson's Rule and Adaptive Simpson's Rule	
Feb 27, Wed	5.4	Gaussian Quadrature Formulas	Final Topic
Mar 1, Fri	6.1	First-Degree and Second-Degree Splines	
Mar 4, Mon	6.2	Natural Cubic Splines	
Mar 6, Wed		**Midterm Exam**: Chapters 1–6	
Mar 8, Fri	No Class	*Quad Finals*	
Mar 11, Mon	No Class	*Spring Break*	
Mar 13, Wed	No Class	*Spring Break*	
Mar 15, Fri	No Class	*Spring Break*	
Mar 18, Mon	7.1	Taylor Series Methods	
Mar 20, Wed	7.2	Runge-Kutta Methods	
Mar 22, Fri	7.3	Adaptive Runge-Kutta Method	
Mar 25, Mon	7.3	Multistep Methods	Outline due
Mar 27, Wed	7.4	Methods for First and Higher Order Systems	
Mar 29, Fri	8.1	Matrix Factorizations	
Apr 1, Mon	8.4	Iterative Solutions of Linear Systems	
Apr 3, Wed	9.1	Method of Least Squares	
Apr 5, Fri	9.2	Orthogonal Systems and Chebyshev Polynomials	
Apr 8, Mon	9.3	Other Examples of the Least-Squares Principle	
Apr 10, Wed	10.1	Random Numbers	
Apr 12, Fri	10.2	Est. of Areas and Volumes by Monte Carlo Techniques	
Apr 15, Mon	10.3	Simulation	
Apr 17, Wed	11.1	Shooting Method	

Date	***Section(s)***	***Topic***	***Project***
Apr 19, Fri	No Class	*Good Friday*	
Apr 22, Mon	No Class	*Easter Travel*	
Apr 24, Wed	11.2	A Discretization Method	Paper due
Apr 26, Fri	Intro to 12	Partial Differential Equations and Finite Differences	
Apr 29, Mon	12.1	Parabolic Problems	
May 1, Wed	12.2	Hyperbolic Problems	
May 3, Fri	12.3	Elliptic Problems	
May 6, Mon		Student Presentations	
May 8, Wed		Student Presentations	
May 15, Wed		**Final exam**: 12:00-2:00 p.m.	Revision due