MAT122 Exam 1 Review Bee

Department of Mathematics and Computer Science
Gordon College

February 13, 2019

Outline

Part 1: Do-at-the-board problems

- An individual from each team will work out an integral problem at the board.
- Team members will come to the board in order; decide on the the order for your team before we start.
- At each turn an integral (indefinite or definite) will be displayed and the contestant will have 60 seconds to evaluate it.
- The first contestant to correctly finish the problem will earn 2 points for their team.
- Warning: Calling out to help your teammate may cause your team to lose points!

Ready...

Here we go...

$$\int e^{-3t}\,dt$$

$$\int e^{-3t}\,dt$$

$$= -\frac{1}{3}e^{-3t} + C$$

$$\int \frac{x^2 + 4}{x}\,dx$$

$$\int \frac{x^2+4}{x}\,dx$$

$$= \frac{x^2}{2} + 4\ln|x| + C$$

$$\int 2\sin 4x\, dx$$

$$\int 2\sin 4x\, dx$$

$$= -\frac{1}{2}\cos 4x + C$$

(Let $u = 4x$ so $du = 4dx$.)

$$\int \left(\frac{4}{x} + \frac{5}{x^2} \right) \, dx$$

$$\int \left(\frac{4}{x} + \frac{5}{x^2}\right)\, dx$$

$$= 4 \ln |x| - \frac{5}{x} + C$$

$$\int \frac{x}{x^2 + 4}\, dx$$

$$\int \frac{x}{x^2+4}\,dx$$

$$= \frac{1}{2}\ln|x^2+4| + C$$

(Let $u = x^2 + 4$ so $du = 2xdx$.)

$$\int 4x \sec x^2 \tan x^2 \, dx$$

$$\int 4x \sec x^2 \tan x^2 \, dx$$

$$= 2 \sec x^2 + C$$

(Let $u = x^2$ so $du = 2xdx$.)

$$\int \frac{e^{1/x}}{x^2}\,dx$$

$$\int \frac{e^{1/x}}{x^2}\,dx$$

$$= -e^{1/x} + C$$

(Let $u = 1/x$ so $du = \frac{-1}{x^2}dx$.)

$$\int_0^2 (x^2 - 2)\, dx$$

$$\int_0^2 (x^2 - 2)\, dx$$

$$= \left[\frac{x^3}{3} - 2x\right]_0^2 = \frac{8}{3} - 4$$

$$= -\frac{4}{3}$$

$$\int_1^2 \frac{ln x}{x}\, dx$$

$$\int_1^2 \frac{lnx}{x}\,dx$$

Let $u = \ln x$ then $du = (1/x)dx$.
$u(1) = 0,\ u(2) = \ln 2.$

$$\int_0^{\ln 2} u\,du = \left.\frac{u^2}{2}\right|_0^{\ln 2} = \frac{(ln2)^2}{2}$$

$$\int_{-1}^{1} (x^3 - 2x)\, dx$$

$$\int_{-1}^{1} (x^3 - 2x)\, dx$$

$$= 0$$

(Integrand is an odd function; integral of an odd function over an interval symmetric about the y-axis is always zero.)

$$\int_1^2 x\sqrt{x^2+1}\,dx$$

$$\int_1^2 x\sqrt{x^2+1}\,dx$$

Let $u = x^2 + 1$. Then $du = 2xdx$.
$u(1) = 2$, $u(2) = 5$.

$$\frac{1}{2}\int_2^5 u^{1/2}\,du = \frac{1}{3}\left[5^{3/2} - 2^{3/2}\right] = \frac{5\sqrt{5} - 2\sqrt{2}}{3}$$

End of Part 1

That's all!

Part 2: Work-as-a-team problems

- When a question is displayed, work with your team to determine the complete correct answer.
- Raise your hand when you have your answer.
- The first team to finish each problem will send a team-member to the board to write their solution.
- If correct, the team gets 2 points. If incorrect the team loses a point.
- If a team is incorrect, the second team to finish can send someone to the board – if their solution is correct they will get one point.
- No team member may return to the board until all team members have been up.

Ready...

Here we go...

Suppose $y = f(x)$ is given by

x	0	1	2	3	4	5	6
y	2.1	2.5	2.7	3	3.1	2.5	2.2

Use the **Midpoint Rule** with $n = 3$ to estimate the value of $\int_0^6 f(x)\,dx$.

Suppose $y = f(x)$ is given by

x	0	1	2	3	4	5	6
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Use the **Midpoint Rule** with $n = 3$ to estimate the value of $\int_0^6 f(x)\,dx$.

With $n = 3$ we have $\Delta x = (6 - 0)/3 = 2$.

$$\int_0^6 f(x)\,dx \approx [2.5 + 3 + 2.5]\,(2) = 16$$

Find the area between the curves $y = x^3$ and $y = 2x^2 - x$.

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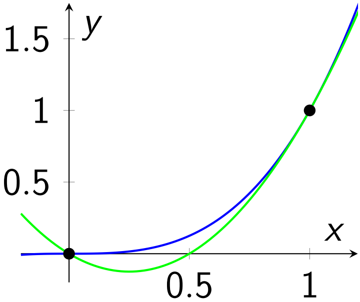

Slicing vertically we have

$$A = \int_0^1 \left(x^3 - (2x^2 - x)\right)\, dx$$
$$= \int_0^1 \left(x^3 - 2x^2 + x\right)\, dx$$
$$= \frac{x^4}{4} - \frac{2x^3}{3} + \frac{x^2}{2}\Big|_0^1$$
$$= \frac{1}{4} - \frac{2}{3} + \frac{1}{2} = \frac{1}{12}.$$

Suppose $y = f(x)$ is given by

x	0	1	2	3	4	5	6
y	2.1	2.5	2.7	3	3.1	2.5	2.2

Use the **Trapezoid Rule** with $n = 3$ to estimate the value of $\int_0^6 f(x)\,dx$.

Suppose $y = f(x)$ is given by

x	0	1	2	3	4	5	6
y	2.1	2.5	2.7	3	3.1	2.5	2.2

Use the **Trapezoid Rule** with $n = 3$ to estimate the value of $\int_0^6 f(x)\,dx$.

$$\int_0^6 f(x)\,dx \approx \frac{6-0}{2\cdot 3}\left[2.1 + (2)(2.7) + (2)(3.1) + 2.2\right]$$
$$= 15.9$$

Set up the integral that computes the volume that results when the region bounded by $y = x^2$ and $y = 2 - x^2$ is revolved about $x = 2$.

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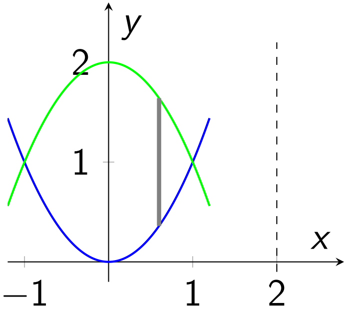

Slicing vertically we have

$$V = \int_{-1}^{1} 2\pi(2 - x)\left[(2 - x^2) - x^2\right]\,dx$$
$$= \int_{-1}^{1} 2\pi(2 - x)\left(2 - 2x^2\right)\,dx$$

Set up the integral that computes the volume that results when the region bounded by $x = y^2$ and $x = 2 + y$ is revolved about
(a) $x = -1$ (b) $y = -1$.

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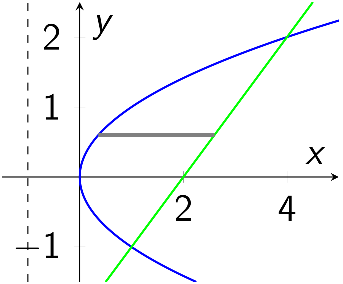
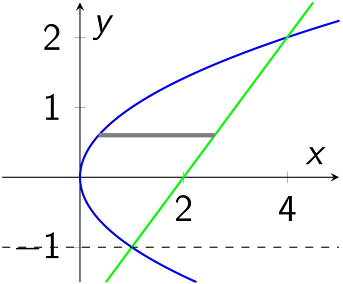


(a)

$$V = \int_{-1}^{2} \pi \left[((2+y) - (-1))^2 - (y^2 - (-1))^2 \right] \, dy$$

$$= \int_{-1}^{2} \pi \left[(3+y)^2 - (y^2+1)^2 \right] \, dy$$

Set up the integral that computes the volume that results when the region bounded by $x = y^2$ and $x = 2 + y$ is revolved about (a) $x = -1$ (b) $y = -1$.



(b)

$$V = \int_{-1}^{2} 2\pi(y - (-1))((2 + y) - y^2)\, dy$$
$$= \int_{-1}^{2} 2\pi(y + 1)(2 + y - y^2)\, dy$$

Set up the integral that computes the volume that results when the region bounded by $y = \cos x$ and $y = 1$ on $[0, 2\pi]$ is rotated about

(a) $x = 0$ (b) $y = 2$.

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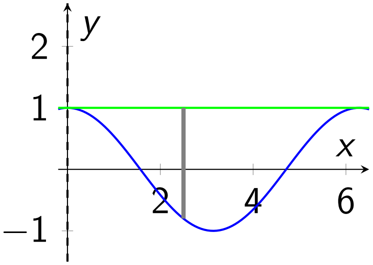
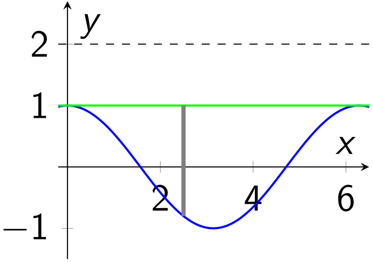


(a)

$$V = \int_0^{2\pi} 2\pi x(1 - \cos x)\, dx$$

Set up the integral that computes the volume that results when the region bounded by $y = \cos x$ and $y = 1$ on $[0, 2\pi]$ is rotated about

(a) $x = 0$ (b) $y = 2$.



(b)

$$V = \int_0^{2\pi} \pi \left[(2 - \cos x)^2 - (2 - 1)^2 \right] \, dx$$

$$= \int_0^{2\pi} \pi \left[(2 - \cos x)^2 - 1 \right] \, dx$$

Suppose $y = f(x)$ is given by

x	0	1	2	3	4	5	6
y	2.1	2.5	2.7	3	3.1	2.5	2.2

Use **Simpson's Rule** with $n = 6$ to estimate the value of $\int_0^6 f(x)\,dx$.

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Use **Simpson's Rule** with $n = 6$ to estimate the value of $\int_0^6 f(x)\,dx$.

$$\int_0^6 f(x)\,dx \approx \frac{6-0}{3\cdot 6}[2.1 + (4)(2.5) + (2)(2.7) + (4)(3) + (2)(3.1) + (4)(2.5) + 2.2] = 15.967$$