

Errors

Thm Suppose f'' is continuous on $[a, b]$ and that $|f''(x)| \leq K$ for all x in $[a, b]$. Then

$$\text{TRAP} \quad |ET_n| \leq K \frac{(b-a)^3}{12n^2} = K \frac{b-a}{12} (\Delta x)^2$$

$$\text{MID} \quad |EM_n| \leq K \frac{(b-a)^3}{24n^2} = K \frac{b-a}{24} (\Delta x)^2$$

Ex How many intervals are required to evaluate

$$\int_1^2 \sin x \, dx \quad \text{to an accuracy of } 10^{-5} \text{ using}$$

The Trapezoid Rule?

$$f(x) = \sin x \quad \text{so} \quad f'(x) = \cos x, \quad f''(x) = -\sin x$$

Notice that $\max_{[1,2]} |-\sin x| = 1$ so we use $K=1$.

$$\text{Need } |ET_n| \leq 10^{-5} \quad \text{so} \quad K \frac{(b-a)^3}{12n^2} = (1) \frac{(2-1)^3}{12n^2} \leq 10^{-5}$$

$$\frac{1}{12n^2} \leq 10^{-5}$$

$$12n^2 \geq 10^5$$

$$n^2 \geq \frac{10^5}{12}$$

$$n \geq \sqrt{\frac{10^5}{12}}$$

$$n \geq 91.287$$

$$\text{Need } n \geq 92$$

to ensure an

accuracy of 10^{-5}

using Trapezoid Rule

Numerical Integration

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Then Suppose $f^{(4)}$ is continuous on $[a, b]$ and that $|f^{(4)}(x)| \leq L$ for all x in $[a, b]$. Then

$$\text{SIMP} \quad |ES_n| \leq L \frac{(b-a)^5}{180 n^4} = L \frac{b-a}{180} (\Delta x)^4$$

Ex Repeat last example using Simpson's Rule

$$f(x) = \sin x \rightarrow f^{(4)}(x) = \sin x \quad \begin{array}{l} \max_{[1,2]} |\sin x| = 1 \\ \text{so } L = 1 \end{array}$$

$$\text{Need } |ES_n| \leq 10^{-5} \quad \text{so} \quad (1) \frac{(2-1)^5}{180 n^4} \leq 10^{-5}$$

$$180 n^4 \geq 10^5$$

$$n^4 \geq \frac{10^5}{180}$$

$$n \geq \sqrt[4]{\frac{10^5}{180}}$$

$$n \geq 4.855$$

We need $n \geq 5$ to ensure an accuracy of 10^{-5} using Simpson's Rule