Surfaces in Space

This section focuses on sketching surfaces in 3D.

Some will find this relatively easy and enjoyable, others may find it frustrating.

I believe, however, that each of you can learn to sketch these surfaces sufficiently well so that you understand the shape and will be able to solve calculus problems involving these surfaces.

Software may be helpful

- Calculators
- Sage
- grapher (on Mac)
- commercial programs

Our focus will be on sketching by hand.

1. Take your time
2. Use pencil and have a good eraser (you can "ink-over" if you want)
3. Try a sketch on scrap paper
4. Remember – the goal is not to produce a piece of art, but for you to understand the shape of the surface.

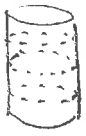
Pay careful attention to text, esp. table on pg 750, 751

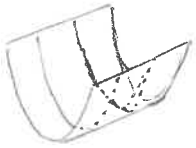
Surfaces in Space

A <u>surface</u> can be thought of as a "skin" or "sheet" It is a two-dimensional object existing in three-dimensional space.

A <u>trace</u> on a surface is a line or curve on the surface. A good example is a contour on a topographical map.

A <u>cylinder</u> is a surface whose traces in every plane parallel to a given plane are the same.





A quadric surface is any surface that results from the graph of $ax^2+by^2+cz^2+dxy+eyz+fxz+gx+hy+jz+k=0$ in three-dimensional space.

Ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

Elliptic Paraboloid $z=ax^2+by^2+c$ $(a,b>0)$

Hyperbolic Paraboloid $z=ax^2-by^2+c$ $(a,b>0)$

Cone $z^2=ax^2+by^2$ $(a,b>0)$

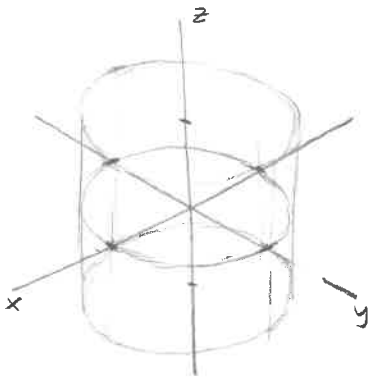
Hyperboloid of one sheet $ax^2+by^2-cz^2=1$ $(a,b,c>0)$

Hyperboloid of two sheets $ax^2-by^2-cz^2=1$ $(a,b,c>0)$

Surfaces in Space

Ex Suppose we want to sketch $x^2 + y^2 = 1$ in 3D.

- this is a circle in the xy-plane
- does not depend on z; circle in any plane of constant z.

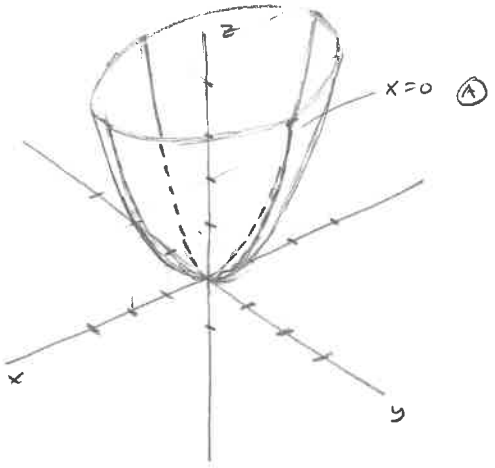


Right Circular Cylinder

① draw and label axes
② determine scale and apply to axes
③ sketch circle in xy-plane
(it will look like an ellipse)
④ Repeat step 3 for differing z values
⑤ Tidy up - erase any unnecessary lines - decide if you want wireframe or surface

Ex Sketch $z = \frac{x^2}{2} + y^2$

① Draw & label axes
② draw "trace" curves -
Ⓐ $x = 0$
Ⓑ $y = 0$
③ draw ellipse at $z = 4$
④ tidy up



Elliptic Paraboloid

Surfaces in Space

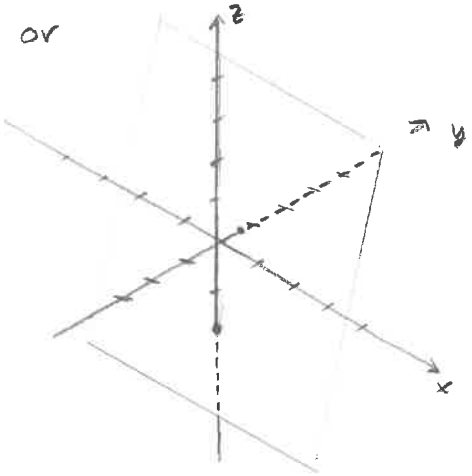
Ex Sketch $3y - z = 2$

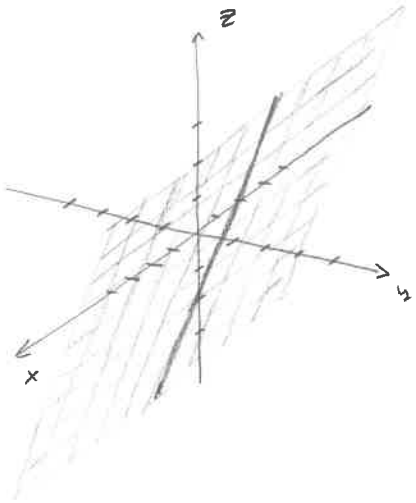
This is independent of x

and we can solve for z

$z = 3y - 2$

or



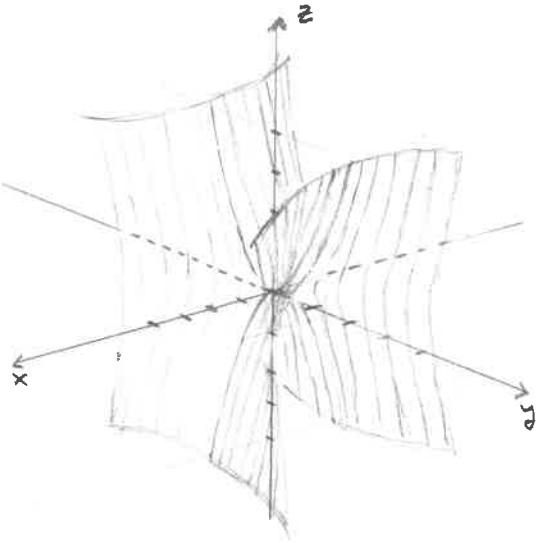


This is a plane

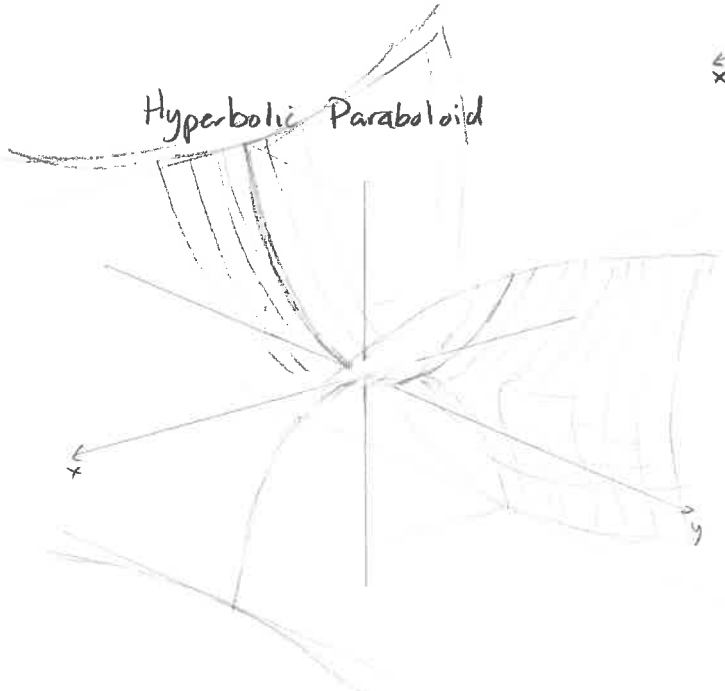
Ex Sketch $z = y^2 - x^2$

$x = 0 \rightarrow z = y^2$

$y = 0 \rightarrow z = -x^2$



Hyperbolic Paraboloid



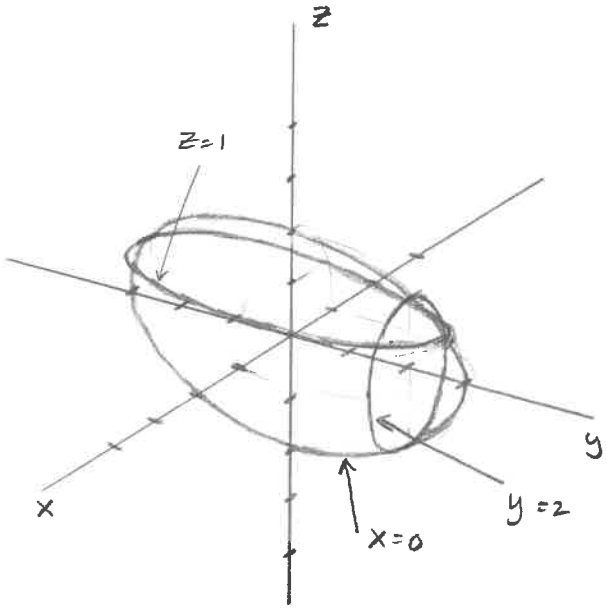
Surfaces in Space

Ex Sketch the following traces for $x^2 + \frac{y^2}{9} + \frac{z^2}{4} = 1$

a) $x = 0$

b) $z = 1$

c) $y = 2$



Ellipsoid

Note:

A <u>cylinder</u> is a surface which, in every plane parallel to a given plane, has the same trace.