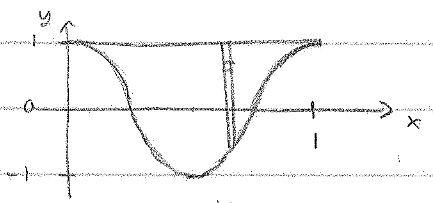
Area, Volume, and Centers of Mass

In Calculus II you found areas and volumes using single integrals.

Double integrals provide a slightly different approach, but yield the same result.

Ex Find the area bounded by $x=0$, $x=1$, $y=\cos 2\pi x$, $y=1$



$$\iint_R dA = \text{area of } R$$

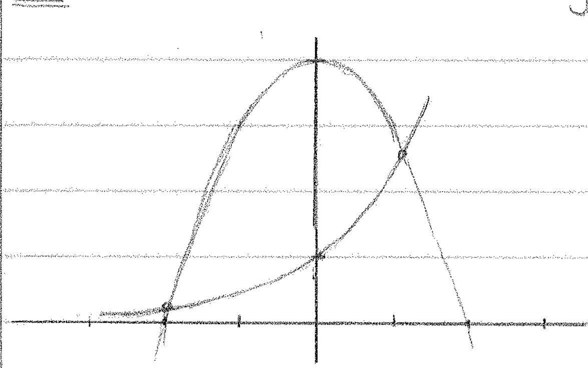
$$\int_0^1 \int_{\cos 2\pi x}^{1} \underbrace{dy\,dx}_{dA} = \int_0^1 y\Big|_{\cos 2\pi x}^{1} dx = \int_0^1 (1-\cos 2\pi x)\,dx$$

$$= x - \frac{1}{2\pi}\sin 2\pi x \Big|_0^1$$

$$= 1 - \frac{1}{2\pi}\sin 2\pi - \left(0 - \frac{1}{2\pi}\sin 0\right)$$

$$= \boxed{1}$$

Ex Find the area between $y=4-x^2$ and $y=e^x$



We will integrate along y first, so the bounds are e^x and $4-x^2$. We need, however, to approximate the points of intersection.

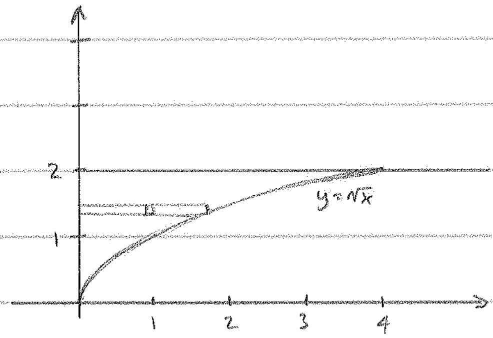
$x=-1.96464$ $\quad$ $x=1.05801$ (use calculator)

$$A = \int_{-1.96464}^{1.05801}\int_{e^x}^{4-x^2} dy\,dx = \int_{-1.96464}^{1.05801}(4-x^2-e^x)\,dx = 4x-\frac{x^3}{3}-e^x\Big|_{-1.96464}^{1.05801}$$

$$= 6.42769$$

Ex Find the volume of the region bounded by

$x=0,\ y=\sqrt{x},\ y=2,\ z=0,\ z=3-xy$



$dv = (3-xy)\,dx\,dy$

$$V = \int_0^2\int_0^{y^2} (3-xy)\,dx\,dy = \int_0^2 3x - \frac{x^2}{2}y\Big|_0^{y^2}\,dy$$

$$= \int_0^2 3y^2 - \frac{y^5}{2}\,dy = y^3 - \frac{y^6}{12}\Big|_0^2 = 8 - \frac{64}{12} = \frac{24-16}{3} = \boxed{\frac{8}{3}}$$